Ballistic spin transport through electronic stub tuners: spin precession, selection, and square-wave transmission

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Ballistic spin transport is studied through electronic tuners with double stubs attached to them. The spins precess due to the spin-orbit interaction. Injected polarized spins can exit the structure polarized in the opposite direction. A nearly square-wave spin transmission, with values 1 and 0, can be obtained using a periodic system of symmetric stubs and changing their length or width. The gaps in the transmission can be widened using *asymmetric* stubs. An additional modulation is obtained upon combining stub structures with different values of the spin-orbit strength.

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The basic principle of a spin transistor, first formulated in Ref. [1] for a waveguide, is that the spin-orbit interaction or Rashba coupling [2], important in narrow-gap semiconductors, makes injected polarized spins precess and leads to a modulation of the current. It's been demonstrated that the strength of this interaction can be tuned by the application of an external gate voltage [3]. In recent years spin polarized transport has attracted considerable attention as it offers a possibility for quantum computation and quantum logic [4]. However, the reported experimental spin polarizations [5] are very low, about 1%, and make the results controversial since they can be attributed to extraneous effects such as the local Hall field and the resistance mismatch [6].

The idea of ballistic spin transport of Ref. [1] relied on the weakness of the spin-orbit coupling. It was recently applied to nanowires by means of a tight-binding analog of the Rashba Hamiltonian [7] thought to be an improvement over a perturbative treatment. A perfect modulation was reported for *weak* coupling. However, the results were tied to a gradual change of the coupling strength over the interaction region, which may be difficult to achieve experimentally, and those for *strong* coupling may be uncertain due to the large strengths used.

The subject of this paper is a transistor-like modulation of a spin current. Motivated by the results of Refs. [1] and [7] and those on electronic [8], [9], [10], and photonic [11] stub tuners, we consider ballistic spin transport through electronic waveguides with double stubs attached to them (Fig. 1 (a)) periodically. The weakness of the spin-orbit coupling is controlled by back gates [3].

Formulation. In the absence of a magnetic field the spin degeneracy of the 2DEG energy bands at $\mathbf{k} \neq 0$ is lifted by the coupling of the electron spin with its orbital motion. This coupling is described by the Hamiltonian

$$H_{so} = \alpha(\vec{\sigma} \times \vec{p})_z/\hbar = i\alpha[\sigma_y \partial/\partial x - \sigma_x \partial/\partial y)]. \qquad (1)$$

Here the y axis is along the waveguide and the x along the stub, cf. Fig. 1 (a). The parameter α measures the strength of the coupling; $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the spin Pauli matrices, and $\vec{p}$ is the momentum operator. The experimental values of α range from about 6×10^{-12} eV m, at electron densities $n = 0.7 \times 10^{12}\,\mathrm{cm}^{-2}$, to 3.0×10^{-11} eV m, at electron densities $n = 2 \times 10^{12}\,\mathrm{cm}^{-2}$.

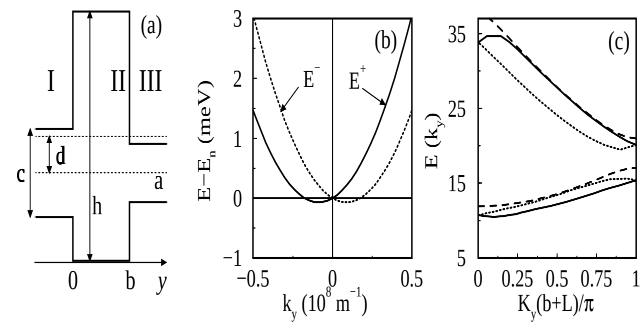

FIG. 1. (a) Schematics of the stub tuner. (b) Dispersion relation for a waveguide based on Eqs. (4) and (5). (c) Superlattice dispersion relation with α equal to zero for the dashed curves and finite for the solid (E^+) and dotted (E^-) curves.

We treat H_{so} as a perturbation. With $\Psi = |k_y, n, \sigma\rangle = e^{ik_y y}\phi_n(x)|\sigma\rangle$ the eigenstate in each region in Fig. 1 (a) the unperturbed states satisfy $H^0|n,\sigma\rangle = E_n^0|n,\sigma\rangle$ with $E_n^0 = E_n + \hbar^2 k_y^2/2m^*$ and $\phi_n(x)$ obeys $[-(\hbar^2/2m^*)d^2/dx^2 + V(x)]\phi_n(x) = E_n\phi_n(x)$, where $V(x)$ is the confining potential assumed to be square-type and high enough that $\phi_n(x)$ vanishes at the boundaries. The perturbed ($H_{so} \neq 0$) eigenfunction, is written as $\sum_{n,\sigma} A_n^\sigma \phi_n(x)|\sigma\rangle$. H_{so} is a 2×2 matrix. Combining it with the 2×2 diagonal matrix H^0 and using $H\Psi = (H^0 + H_{so})\Psi = E\Psi$ leads to the equation

$$\begin{bmatrix} E_m^0 - E & \alpha k_y \\ \alpha k_y & E_m^0 - E \end{bmatrix} \begin{pmatrix} A_m^+ \\ A_m^- \end{pmatrix} = 0; \qquad (2)$$

the resulting eigenvalues $E \equiv E^\pm(k_y)$, plotted in Fig. 1 (b), are

$$E^\pm(k_y) = E_n + (\hbar^2/2m^*)k_y^2 \pm \alpha k_y. \qquad (3)$$

b)

The eigenvectors corresponding to E^+, E^- satisfy $A_m^{\pm} = \pm A_m^{\mp}$. Accordingly, the spin eigenfunctions are taken as $|\pm\rangle = \left(\begin{smallmatrix}1\\ \pm 1\end{smallmatrix}\right)/\sqrt{2}$. For the same energy the difference in wave vectors k_y^+ and k_y^- for the two spin orientations is

$$k_y^- - k_y^+ = 2m^*\alpha/\hbar^2 = \delta. \tag{4}$$

The dispersion relation $E^{\pm}(k_y)$ vs k_y resulting from Eq. (3) is shown in Fig. 1 (b). For the same energy E there are four k_y values and a phase shift δ between the positive or negative k_y^+ and k_y^- values of the branches E^+ and E^-.

The procedure outlined above applies to all regions, I, II, III , in Fig. 1 (a). In each region we have $\phi_n(x) = \sin(n\pi(x + w/2)/w)$, where w is the width of the region along x. Including spin and referring to Fig. 1 (b) we can write the eigenfunction ϕ_1 of energy E in region I as

$$\phi_1 = \sum_m \{a_{1m}^+ e^{i\beta_m y}\begin{pmatrix}1\\1\end{pmatrix} + a_{1m}^- e^{i(\beta_m+\delta)y}\begin{pmatrix}1\\-1\end{pmatrix} + b_{1m}^+$$
$$\times\, e^{-i(\beta_m+\delta)y}\begin{pmatrix}1\\1\end{pmatrix} + b_{1m}^- e^{-i\beta_m y}\begin{pmatrix}1\\-1\end{pmatrix}\}\sin(c_m(x+\frac{c}{2}) \tag{5}$$

Here $c_m = m\pi/c$ and $\beta_m = (2m^*E - c_m^2)^{1/2}$. In region III ϕ_2 is given by Eq. (5) with the changes $1m \to 2m, c \to a$, and $y \to y - b$. In the stub region II, Eq. (5) remains valid with the changes $c \to h$ and $x + c/2 \to x + h/2 - d$.

We now match the wave function and its derivative at $y = 0$ and $y = b$. In this way we can connect the incident waves (to the left of region I) with the outgoing ones (to the right of region III) via a transfer matrix $\hat{M}$

$$\begin{pmatrix} a_{in}^+ \\ a_{in}^- \\ b_{in}^+ \\ b_{in}^- \end{pmatrix} = \hat{M} \begin{pmatrix} a_{out}^+ \\ a_{out}^- \\ b_{out}^+ \\ b_{out}^- \end{pmatrix}. \tag{6}$$

If we connect a spin polarizer (analyzer) to the left (right) of the structure, we can inject electrons and detect the polarization of the outgoing electrons. For spin-up electrons injected into a simple waveguide, where the transmission is always unity, the probability of detecting a spin-down $\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$ electron, after a distance l, will be proportional to $|\langle\left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)|\psi\rangle|^2 = \sin^2(\Delta\theta/2)$ [1,7], where $\Delta\theta = \delta l$ is the phase difference between the up $|+\rangle$ and down $|-\rangle$ spin modes Here we show that by attaching stubs the transmission can be modulated more efficiently: we can flip the spin of the incident electrons, or block it completely, and thus establish a spin transistor.

Results. We consider electrons of energy $E = 48$meV injected into a $In_{0.53}Ga_{0.47}As$ multi-stub structure, with $m^* = 0.042m_0$ and $\alpha = 1.6 \times 10^{-11}$eVm. The parameters are $c = a = 250$Å, $b = 150$Å, and $h = 1859$Å; the length of the waveguide segment between two neighbouring stubs is $L = 207.5$Å. To verify the validity of the perturbation theory we evaluated the bound states of an isolated unit made of one stub and one waveguide segment with the same values for c, a, b and L. The ratio (intersubband mixing energy/difference between lowest two bound states) is less than 10%. In Fig. 1 (c) we show the first and second energy bands of a superlattice made of such units without (dashed curves) and with (solid and dotted curves) spin-orbit coupling. The energy difference due to the latter is about 15% of the electron energy.

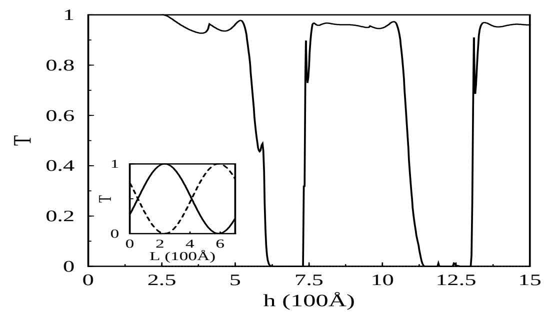

FIG. 2. Transmission T vs stub length h. The inset shows T^- (solid curve) and T^- (dotted curve) electrons, normalized to the input flux T^+, vs the (*stubless*) waveguide's length h.

We now consider the possibility of spin-transistor action using *symmetric* stubs. The transmission plotted in Fig. 2 shows the possibility of spin blocking as well as spin flipping for a periodic array of fifteen stubs arranged in three groups. The first five stubs form the first group and so on. The parameters $c = a = 250$Å, $L = 267.5$Å, and h are the same for all groups but α and b differ from one group to another: we took $\alpha_1 = 1.05\alpha_2 = 1.1\alpha_3 = 1.6\times10^{-11}$eVm and $b_1 = 0.95b_2 = 0.9b_3 = 150$Å. Because the output spin orientation depends on the total length of the device, we choose the length for which complete spin flip occurs through the *stubless* device; then we can control the transmission by adjusting h. As shown in the figure, we can completely block the exit of electrons of either spin orientation for h in the ranges of the gaps ($T = 0$), i.e., for $620\mathring{A} \leq h \leq 680\mathring{A}$ and $1130\mathring{A} \leq h \leq 1285\mathring{A}$.

A better control of the transmission can be obtained if we employ *asymmetric* stubs. For the results shown in Fig. 3 we inject spin-up electrons of energy $E = 48meV$ into a five-identical-unit device with $c = a = 250$Å,

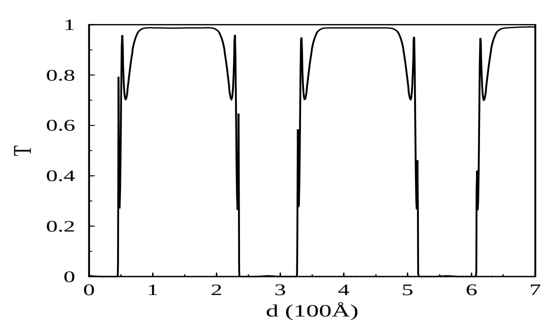

FIG. 3. Transmission T vs asymmetry parameter d.

$b = 150$Å, $L = 207.5$Å, and $h = 1859$Å for all units. L is chosen so that only spin-down electrons appear in the output and h is chosen such that all electrons are

reflected ($T = 0$) when the stubs are symmetric. As realized, by shifting all stubs by a distance d, cf. Fig. 1 (a), with the help of side gates, we observe a nearly perfect spin-transistor behavior: the transmission jumps almost from 0 to 1 and only spin-down electrons come out though only spin-up electrons are injected.

Discussion. A qualitative understanding of the results shown in Figs. 2 and 3 is easily reached if we combine the spin precession in a single waveguide [1], due to the spin-orbit coupling, with the basic idea of a stub tuner [12] and its refinements [9]. In a stub tuner waves reflected from the walls of the stub, where the wave function vanishes, may interfere constructively or destructively with those in the main waveguide and result, respectively, in an increase or decrease of the transmission. Refining this idea, it was shown in Ref. [9] that using *asymmetric* double stubs the transmission of *spinless* electrons could be blocked completely. Combining several stubs would result in a nearly square-wave transmission as a function of the asymmetry parameter d. The transmission shown in Figs. 2 and 3 is simply the result of this behavior when combined with the spin precession due to the spin-orbit coupling since the length of the device was chosen such that spin flip would occur in the stubless waveguide.

An important question is how robust the results are if we change any of the stub parameters. As shown in Figs. 2 and 3 the transmission is not always perfect: near the edges of the gaps we have peaks less high than unity and their number increases if we change, e.g., h in Fig. 2 or d in Fig. 3. However, the gaps are wide and one can widen them further by adjusting b, c, and especially the strength α that can be controlled by a back gate.

Another question is the influence of the stub shape on the transmission output. But as in electronic stub tuners [9] , here two stubs of different shape do not change the transmission qualitatively. We break each shape in a

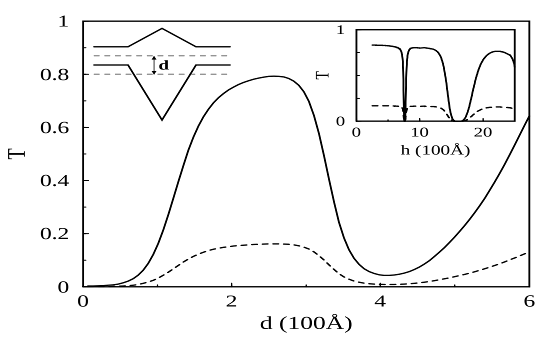

FIG. 4. Transmission T through two *asymmetric*, triangular double stubs vs asymmetry parameter d. The solid (dotted) curve is for spin-down (spin-up) electrons. The right inset shows T vs h through two *symmetric*, triangular stubs.

series of rectangular segments with the same width b_i and different heights. Each segment is described by a transfer matrix M_i. The full shape is well described by the product $M^{tot} = \Pi_i M_i$ if b_i is much smaller than the electronic wavelength. In Fig. 4 we consider a structure with two *asymmetric* triangular double stubs, as shown in the left inset, and plot T vs d, normalized to the input spin-up T^+ and further specified in the caption, for $b = 377$Å, $h = 1660$Å, $L = 275$Å, and $a = 250$Å. The total length l $(= 2b + 2L)$ is chosen such that neither T^+ nor T^- is completely suppressed. If l or α is chosen such that, e. g., T^+ is completely suppressed, then T^- is given approximately by the sum of the solid and dotted curves in Fig. 4 and resembles closely that of Fig. 3. By comparing Fig. 2 with the right inset of Fig. 4 and Fig. 3 with Fig. 4 we see that the qualitative behavior for stubs of different shapes is the same. The shape affects mainly the period of the transmission when we combine several stubs, compare Figs. 2-4.

All the results presented so far are valid when only a single mode propagates in the waveguide. If more modes are allowed to propagate the transmission pattern becomes more complex but it is still possible to have a periodic transmission output, e.g., as in Fig. 2, if b is short enough that only a single mode can penetrate into the stub region [9]. Details will be given elsewhere.

In summary, we combined the spin precession in a waveguide, due to the spin-orbit coupling, with the basic physics of a stub tuner, and applied it to the transmission through several stubs. We showed that we can select the spin of the outgoing electrons to be the same as or opposite to that of the injected spin-polarized electrons. More important, we can have a nearly binary square-wave transmission (spin-valve effect) for either spin orientation. In this respect, *asymmetric* stubs, with shape controlled by lateral gates [8], give the best results.

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